The Nusselt number associated with this heat-transfer rate is defined as

$$Nu = \frac{Q}{kL(T_1 - T_2)} = \frac{2\pi}{Ra_L\left(\frac{R}{L}\right)^2}\left(\int_0^1 ruT\,dr\right)_{x=1}. \quad (49)$$

For heat transfer in the similarity regime, equations (18), (19) and (25) were substituted in equation (49) to yield

$$Nu = 0.255\,Ra_L\left(\frac{R}{L}\right)^2. \quad (50)$$

This result appears as a straight line on Fig. 2, in the range $Ra_L(R/L)^2 \leq 11.81$ where the similarity regime exists.

For the boundary-layer regime we developed two solutions. The Nusselt number calculated from the integral solution is shown on Fig. 2 in the range $(R/L)^2 Ra_L \geq 218.83$. Using the calculus of limits, one can show that as $(R/L)^2 Ra_L$ approaches infinity, the Nusselt number is given by

$$Nu = 5.62\frac{R}{L}Ra_L^{1/2}. \quad (51)$$

The Nu result based on the Oseen-linearized boundary-layer solution was also plotted on Fig. 2 in the range $(R/L)^2 Ra_L \geq 768$. In the high Ra_L limit we obtain:

$$Nu = 2\pi\frac{R}{L}Ra_L^{1/2}. \quad (52)$$

Since in the high Rayleigh-number limit the free convection phenomenon inside the well approaches free convection along a flat vertical surface, it is possible to compare results (51) and (52) with the Nusselt number obtained by Cheng and Minkowycz [3] for a vertical plate,

$$Nu = 5.58\frac{R}{L}Ra_L^{1/2}. \quad (53)$$

Comparing asymptotes (51) and (52) with asymptote (53) we find excellent agreement for both boundary-layer solutions developed in this paper. In particular, the integral boundary-layer solution appears to be the better of the two, its asymptotic Nu differing by only 0.7% from the result of Cheng and Minkowycz [3]. This comparison supports the validity of the Karman–Pohlhausen integral method used in this paper to analyze not only the boundary layer regime but also the similarity regime.

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A NOTE ON KUTATELADZE'S EQUATION FOR PARTIAL NUCLEATE BOILING*

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NOMENCLATURE

- h, heat-transfer coefficient in partial nucleate boiling; $h = q/(t_w - t_f)$;
- h_f, one-phase forced-convection heat-transfer coefficient;
- h_b, heat-transfer coefficient during developed boiling [see equation (2) or (3)];
- q, specific heat flux;
- q_b, saturated pool boiling heat-flux corresponding to temperature t_w;
- t_f, bulk temperature;
- t_s, saturation temperature;
- t_w, wall temperature;
- $t_{w,b}$, saturated pool boiling wall temperature corresponding to the heat flux q;
- w, velocity.

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In the transition region from forced convection to nucleate boiling, the heat-transfer coefficient is affected by the flow velocity and degree of subcooling. This influence is observed with fluids flowing in tubes or annular spaces and for fluids flowing normally to horizontal cylinders.

In order to evaluate the effect of fluid velocity on the surface boiling heat transfer in tubes at saturation conditions, Kutateladze [1] proposed the following relationship:

$$h/h_f = \left[1 + \left(\frac{h_b}{h_f}\right)^n\right]^{1/n} \quad (1)$$

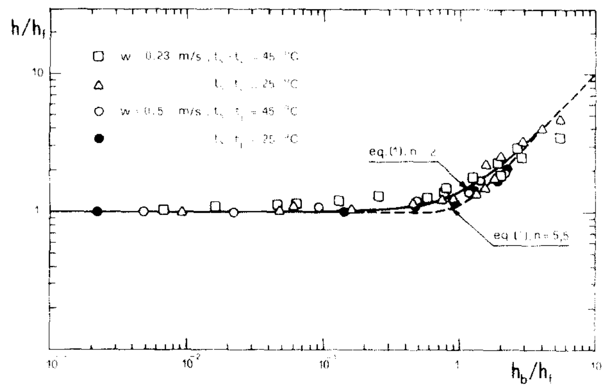

FIG. 1. Comparison of equation (1) with experimental data: h_b evaluated using relation (2).

where h_f and h_b are the heat-transfer coefficients during one-phase forced convection and during developed nucleate boiling, respectively. For practical calculations, the same author obtained a numerical value of $n = 2$, for the exponent by processing the Sterman's test results.

Equation (1) has been extended to subcooled boiling conditions by Tong [2] and by Fand *et al.* [3], defining the heat-transfer coefficient h_b in different ways.

Tong suggested:

$$h_b = q_b/(t_w - t_f) \tag{2}$$

where q_b is the heat flux taken from saturated pool boiling data corresponding to the effective wall temperature t_w.

Fand *et al.* proposed:

$$h_b = q/(t_{w,b} - t_f) \tag{3}$$

where $t_{w,b}$ is the wall temperature taken from saturated pool boiling data at the effective heat flux q. The heat-transfer coefficient h has been assumed by both the authors as

$$h = \frac{q}{t_w - t_f}.$$

Where q and $(t_w - t_f)$ are respectively the effective values of the heat flux and temperature difference during partial nucleate boiling.

The two h_b definitions lead to different results as can be easily seen.

In order to evaluate which is the more suitable relationship for correlating the experimental data, a comparison between these two definitions, using our experimental results, is given in this communication. These experimental data obtained in a previous work [4] concern saturated pool boiling and forced-convection boiling experiments, using the same nickel test section; the coolant was demineralized water at 1.21 bar pressure. The saturated pool boiling experiments was carried out to obtain the q_b and $t_{w,b}$ values for the equations (2) and (3).

In Fig. 1 the h/h_f ratio of heat-transfer coefficients for partial nucleate boiling h and for single-phase forced convection h_f, is presented vs the h_b/h_f ratio, where h_b is evaluated using the relation (2).

It is apparent that equation (1) sufficiently fits our data when the exponent $n = 2$, suggested by Kutateladze, is considered.

Our experimental data are also reported in Fig. 2, evaluating h_b by means of Fand's definition.

Equation (1) fits our data with a good accuracy if a higher n value is assumed equal to 5.5. The same n value was found by Fand *et al.* to correlate their experimental data during simultaneous boiling and forced convection from a horizontal cylinder to water in cross flow.

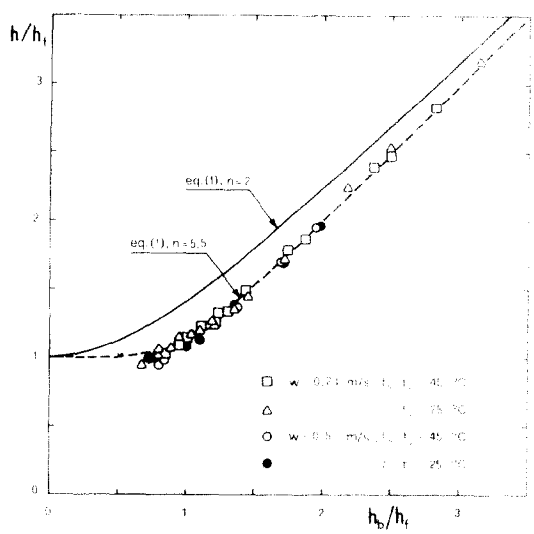

FIG. 2. Comparison of equation (1) with experimental data: h_b evaluated using relation (3).

In conclusion, Kutateladze's equation (1) with $n = 2$, is suitable for correlating experimental data under saturated and subcooled conditions, provided that h_b is defined according to relation (2). When h_b is defined with equation (3), a higher n value for the exponent is required. The latter procedure allows our data to fit with a better accuracy.

Moreover, since the effective heat flux q is normally known for design calculations, equation (3) would seem to be more suitable; the temperature $t_{w,b}$ can then be found and h_b determined directly. Equation (2) instead requires an iterative approach for evaluating the heat-transfer coefficient h_b.

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